CATHOLIC EDUCATION WESTERN AUSTRALIA

ViSN Virtual School Network

2020 Year 11 ViSN Mathematics Specialist Unit 1 & 2
Test 1 – Combinatorics & Proof
Section One – Calculator Free

Mr Daniel Comtesse
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Calculator Free:______/17
Calculator Assumed:_____/27

Result: _____/44 ______%

Student Name: Solutions,

School: ______________________

Time allowed: Section One - 15 minutes
Section Two – 30 minutes

Assessment Date: Tuesday Week 4, Term 1 – 25/02

Material required/recommended

To be provided by the supervisor
This Question/Answer Paper
SCSA Formula Sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Submission Details
Timed Assessments are to be returned to the ViSN teacher by the ViSN mentor (scan completed assessment and email to teacher above) within 24 hours of assessment date (above).

Instructions to Students

1. **ALL** questions should be attempted.

2. Write your answers in the spaces provided in this Question/Answer Booklet.

3. **SHOW ALL YOUR WORKING CLEARLY**. Your working should be sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Correct answers given without supporting reasoning may not be allocated full marks. Incorrect answers given without supporting reasoning cannot be allocated any marks.

4. If you repeat an answer to any question, ensure that you cancel the answers you do not wish to have marked.

5. It is recommended that you **do not use pencil**, except in diagrams.

Question 1 **[2, 2, 2, 2 = 8 marks]**

(a) Write the inverse of the following true statement and comment on the truth of the inverse statement.

"If the discriminant of the quadratic formula is zero, then the quadratic has just one real root."

'If the discriminant of the quadratic formula is not zero, then the quadratic will have no or two real roots.'

Inverse statement is true.

(b) Write the converse of the following true statement and comment of the truth of the converse statement.

"If $x > 3$, then $x > 2$."

If $x > 2$, then $x > 3$

False as if $x = 2.5$, then $x \not> 3$.

(c) Determine the truth of the following statements, using an example or counter-example to support each answer.

(i) If $z \in \mathbb{R}$ and z^3 is an even number, then z is an even number.

If $z^3 = 10$, the $z = \sqrt[3]{10}$ which is not even.
Hence, false.

(ii) If $x, y \in \mathbb{Z}$ and $x > y$, then $x^2 > y^2$.

If $x = 2$, $y = -3$, then $x > y$, but $x^2 \not> y^2$

as $4 \not> 9$.

False.

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Question 2 **[2 marks]**

In a particular football league, there are eight teams. The league is run so that every team plays every other team exactly once and no game ends in a tie.

Use the pigeon hole principle to show that if no team loses all its games, then at least two teams finish the competition with the same number of wins.

Number of wins for each team is a 'pigeonhole':

1, 2, 3, 4, 5, 6, 7

✓ Identifies pigeonholes & pigeons

But there are 8 teams (pigeons).

Therefore two teams must finish on the same number of wins.

✓ concludes.

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Question 3 **[2, 2, 3, = 7 marks]**

It can be shown for all $n \geq 0$,

$$^{n+1}P_r = \frac{n+1}{n-r+1} \times {}^{n}P_r$$

(a) Show that the identity is true when $n = 4$ and $r = 2$.

$LHS = {}^5P_2 = 20$

$RHS = \frac{5}{3} \times {}^4P_2$

$= \frac{5}{3} \times 12$

$= 20$

$= LHS$

Given that ${}^8P_4 = 1\,680$, ${}^{12}P_5 = 95\,040$ and ${}^{12}P_6 = 665\,280$, evaluate using the identity above

(b) ${}^{11}P_6$.

If $n = 11$, $r = 6$

${}^{12}P_6 = \frac{12}{6} \times {}^{11}P_6$

$665\,280 = 2 \times {}^{11}P_6$

$332\,640 = {}^{11}P_6$

(c) ${}^{10}P_4$.

${}^{10}P_4 = \frac{10}{9-4+1} \times \frac{9}{8-4+1} \times {}^8P_4$ uses telescoping.

$= \frac{10}{6} \times \frac{9}{5} \times 1680$

$= 3 \times 1680$

$= 5040$

End of Calculator Free Section

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Additional working space

Question number: _________

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Special Items: Scientific calculator and/or CAS calculator, notes on one A4 (one sided) page.

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Question 4 **[1, 2 = 3 marks]**

(a) Determine the number of different permutations of the letters in the word PARALLEL.

No. of permutations $= \dfrac{8!}{2!\,3!}$

$= 3360$

(b) A password is formed using all seven of the characters $, %, @, Y, Z, 1 and 2 just once. Determine the number of different passwords that are possible in which all the symbols are adjacent, all the letters are adjacent and all the digits are adjacent.

$= 3! \times 3! \times 2! \times 2!$

$= 144$ passwords

Adjacent part correct

3

Question 5 **[4, 5 = 9 marks]**

(a) Determine the number of different four-letter passwords that can be made by arranging a selection of four letters chosen from the list P, Q, R, R, R, R and S.

1 R

$^1C_1 \times {}^3C_3 \times 4! = 24$ ✓Breaks into cases of R

2 R's

$\therefore$ Total $= 73$ ✓Total

$^2C_2 \times {}^3C_2 \times \frac{4!}{2!} = 36$

✓one case correct

✓all cases correct

3 R's

$^3C_3 \times {}^3C_1 \times \frac{4!}{3!} = 12$

4 R's

$^4C_4 \times \frac{4!}{4!} = 1$

(b) Determine the number of positive integers between 1 and 240 inclusive that are <u>not</u> divisible by at least one of the integers 4, 5 or 6.

Multiples of $4 = 60$

" $5 = 48$ ✓single sets

" $6 = 40$

Multiples of $20 = 12$

" $12 = 20$ ✓double intersections

" $30 = 8$

Multiples of $60 = 4$ ✓triple intersection

$\therefore$ Total divisible $= 60 + 48 + 40 - 12 - 20 - 8 + 4$

$= 112$ ✓inclusion - exclusion

$\therefore$ Total <u>not</u> divisible $= 240 - 112$

$= 128$ ✓complement.

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Question 6 **[1, 3, 3 = 7 marks]**

Fifteen children at a summer camp are to be divided into two groups of nine and six.

(a) Determine the number of different groupings.

$= {}^{15}C_9$

$= 5005$ ✓

(b) Determine how many groupings are possible if the two youngest children must be in the same group.

Youngest in Larger group

${}^{2}C_2 \times {}^{13}C_7 = 1716$ ✓

$\therefore$ Total $= 2431$ ✓

Youngest in smaller group.

${}^{2}C_2 \times {}^{13}C_4 = 715$ ✓

(c) If ten of the fifteen were girls, in how many of the different groupings do both groups contain more girls than boys?

5 in large

${}^{10}C_5 \times {}^{5}C_4 = 1260$

✓ Determines possible groups

6 in large

${}^{10}C_6 \times {}^{5}C_3 = 2100$

✓ Calcs 1 possible group correctly

$\therefore$ Total $= 3360$

✓ Second group & total

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Question 7 **[2, 2 = 4 marks]**

A quadrilateral is to be formed using the dots below as vertices.

(a) How many quadrilaterals can be formed?

$^6C_2 \times {}^5C_2 = 150$

✓uses combinations

✓correct

(b) Given the horizontal spaces are all equal, how many of the quadrilaterals are parallelograms?

$= 5\times4 + 4\times3 + 3\times2 + 2\times1$ ✓

$= 40$ ✓

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Question 8 **[4 marks]**

Prove by contradiction that, for every positive real number x, $\frac{x}{x+1} < \frac{x+1}{x+2}$.

Assume that $\frac{x}{x+1} > \frac{x+1}{x+2}$ ✓ Assumption

$x(x+2) > (x+1)^2$ ✓ rearranges

$x^2 + 2x > x^2 + 2x + 1$

$0 > 1$

✓ $0 > 1$

This is a contradiction as $0 \ngtr 1$.

Hence, $\frac{x}{x+1} < \frac{x+1}{x+2}$ for positive, real x. QED. ✓ concludes.

End of Assessment

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Additional working space

Question number: _________